BC Calc Vector 3 Name: Fr Chris Block: Seat:

1. (No Calc) The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$, $y(t) = \frac{2}{3}t^3$.

   (a) Find the magnitude of the velocity vector at $t = 2$.

$$\vec{v}(t) = \langle 2t, 2t^2 \rangle$$

$$\vec{v}(2) = \langle 4, 8 \rangle$$

$$\|\vec{v}(2)\| = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$$

   (b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

$$\int_0^4 \sqrt{(2t)^2 + (2t^2)^2}\, dt$$

$$\approx 46.061$$

or 46.062

   (c) Find $\frac{dy}{dx}$ as a function of x.

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t^2}{2t} = t$$

since $x = t^2 - 2$

$$\frac{dy}{dx} = t = \sqrt{x+2}$$

   (d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

$$\frac{2}{3}t^3 = 0$$

$$t = 0$$

$$\vec{a}(t) = \langle 2, 0 \rangle$$

2. (No Calc) An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with the velocity vector $v(t) = \langle (t+1)^{-1}, 2t \rangle$. At time $t = 1$, the object is at $(\ln 2, 4)$.

(a) Find the position vector.

$$\vec{s}(t) = \int \left\langle \frac{1}{t+1}, 2t \right\rangle dt$$

$$= \langle \ln|t+1| + c_1,\ t^2 + c_2 \rangle$$

$$\vec{s}(1) = \langle \ln 2, 4 \rangle$$

so $c_1 = 0$, $c_2 = 3$

$$\vec{s}(t) = \langle \ln|t+1|,\ t^2+3 \rangle$$

(b) What is the speed of the particle when $t = 1$.

$$\sqrt{\left(\frac{1}{2}\right)^2 + 2^2} = \frac{\sqrt{17}}{2}$$

(c) Write an equation for the line tangent to the curve when $t = 1$.

$$\frac{dy}{dx} = \frac{2t}{\frac{1}{t+1}} = 2t(t+1)\Big|_{t=1} = 4$$

$$y - 4 = 4(x - \ln 2)$$

(d) At what time $t \geq 0$ does the line tangent to the particle at $(x(t), y(t))$ have a slope of 12?

$$2t(t+1) = 12$$

$$2t^2 + 2t - 12 = 0$$

$$2(t+3)(t-2) = 0$$

$t = -3$ or 2

(recall abs value? Both work)

(e) Write an expression that represents how far has the particle travelled from time $t = 0$ to $t = 1$.

$$\int_0^1 \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2}\, dt$$

3. (Calc OK) A particle moving along a curve in the xy-plane has position $(x(t), y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \le t \le 10$.
Find the velocity vector at the time when the particle's vertical position is $y = 7$.

$$\vec{v}(t) = \langle 2 + 3\cos t,\ 2t - 2\sin t \rangle$$

$$y(t) = t^2 + 2\cos t = 7$$

when $t = 2.9964952 = A$

$$\vec{v}(A) = \langle -0.968,\ 5.703 \rangle$$

or $\langle -0.968,\ 5.704 \rangle$

4. (Calc OK) A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\dfrac{dx}{dt} = 1 + \sin(t^3)$. The derivative $\dfrac{dy}{dt}$ is not explicitly given. For any time t, $t \ge 0$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{so} \quad \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

So $$\frac{dy}{dt} = (t+3)(1+\sin t^3)$$

$$\frac{dx^2}{dt^2} = 3t^2 \cos t^3 \Big|_{t=2} = 12\cos 8$$

and $$\frac{dy^2}{dt^2} = (t+3)(3t^2\cos t^3) + (1)(1+\sin t^3)\Big|_{t=2} = 60\cos 8 + 1 + \sin 8$$

Hence $\vec{a}(2) = \langle -1.746,\ -6.740 \rangle$

(or -6.741)

(Calc not really needed here)

5. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dy} = \cos(e^t)$ and $\frac{dy}{dt} = \sin(e^t)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point $(3, 2)$.

(a) Find the equation of the tangent line to the curve at the point where t = 1.

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{\sin e^1}{\cos e^1} = \tan e$$

$$y - 2 = \tan e\,(x-3)$$

$$y - 2 = (-0.4505)(x-3)$$

(b) Find the speed of the object at t = 1.

$$\sqrt{\cos^2 e + \sin^2 e} = 1$$

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

$$\int_0^2 \sqrt{\cos^2 e^t + \sin e^t}\, dt = \int_0^2 dt$$

$$= 2 - 0$$

(d) Find the position of the object at time $t = 2$.

$$x = 3 + \int_1^2 \cos e^t\, dt = 2.8957$$

$$y = 2 + \int_1^2 \sin e^t\, dt = 1.6759$$

6. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \sin(t^3 - t)$ and $\frac{dy}{dt} = \cos(t^3 - t)$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

$\vec{a}(t) = \langle (3t^2-1)\cos(t^3-t), -(3t^2-1)\sin(t^3-t) \rangle$

$\vec{a}(3) = \langle 26 \cos 24, -26 \sin 24 \rangle$

about $\langle 11.028, 23.545 \rangle$

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

$$\frac{dy}{dx} = \left.\frac{\cos(t^3-t)}{\sin(t^3-t)}\right|_{t=3} = \cot 26 \approx .84835$$

$$y = \cot 26 (x-1) + 4$$

(c) Find the magnitude of the velocity vector at $t = 3$.

$$\|\vec{v}\| = \sqrt{\sin^2 26 + \cos^2(26)} = 1$$

(d) Find the position of the particle at time $t = 2$.

$$x = 1 + \int_3^2 \sin(t^3-t)\, dt \approx 0.932$$

$$y = 4 + \int_3^2 \cos(t^3-t)\, dt \approx 4.002$$

7. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dy}{dt} = 2 + \sin(e^t)$. The derivative $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4, 5)$.

   (a) Find the y-coordinate of the position at time $t = 1$.

$$y = 5 + \int_3^1 2 + \sin e^t \, dt = 1.268$$

   (b) At time $t = 3$, the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

$$\frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dx}{dt} = (2 + \sin e^t)\left(-\frac{5}{9}\right)$$

$$\left.\frac{dx}{dt}\right|_{t=3} = (2 + \sin e^3)\left(\frac{-5}{9}\right)$$

$$\approx -1.635$$

   (c) Find the speed of the object at time $t = 3$.

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \approx 3.368$$